Exam Méca - Session 2 - juin 2012

Partie I :

a) $I_{Gz}(C) = \int (x^2+y^2)\,dm = \int r^2 dm = mr^2$

b) $\vec{P} = m\vec{V}_{G/R} = m\dot{x}\,\vec{e_x}$, $\vec{L_G} = I_{Gz}.\dot{\theta}\,\vec{e_z} = mr^2\dot{\theta}\,\vec{e_z}$

Partie II :

a) $\vec{V_g} = \vec{V}_{(I_1 \in C/R)} - \vec{V}_{(I_2 \in sol/R)} = \vec{V}_{(I_1 \in C/R)}$

$= \vec{V}_{(G\in C/R)} + \vec{I_1 G} \wedge \dot{\theta}\vec{e_z} = \dot{x}\,\vec{e_x} + r\vec{e_y} \wedge \dot{\theta}\vec{e_z}$

soit $\boxed{\vec{V_g} = (\dot{x} + r\dot{\theta})\,\vec{e_x}}$, $V_g(t=0) = (V_0 - r\omega_0)\,\vec{e_x}$

b) $\frac{d\vec{p}}{dt} = \sum \vec{f}_{ext} = \vec{P} + \vec{T} + \vec{N}$ et $\left.\frac{d\vec{L_G}}{dt}\right|_R = \vec{\mathcal{M}}_G(\vec{P}+\vec{T}+\vec{N})$

$= \vec{GI} \wedge (\vec{T}+\vec{N})$

c) $\begin{cases} m\ddot{x} = T \\ m\ddot{y} = -mg + N = 0 \end{cases}$ et $mr^2\ddot{\theta}\,\vec{e_z} = -r\vec{e_y}\wedge(T\vec{e_x} + N\vec{e_y}) = rT\vec{e_z}$

$\rightarrow \underline{mr^2\ddot{\theta} = rT} \Rightarrow \ddot{\theta} = \frac{T}{mr}$

- A $t=0$: $V_g \neq 0$ → roulement avec glissement

→ loi de Coulomb $\|\vec{T}\| = \mu\|\vec{N}\|$ avec $V_g = V_0 - r\omega_0 < 0$

↳ $T > 0 \rightarrow \|\vec{T}\| = T$

soit $T = \mu N = \mu mg$ d'où : $\begin{cases} \ddot{x} = \mu g \rightarrow \dot{x} = \mu g t + V_0 \\ \ddot{\theta} = \frac{\mu g}{r} \rightarrow \dot{\theta} = \omega = \frac{\mu g t}{r} - \omega_0 \end{cases}$

Soit alors $V_g = \dot{x} + r\dot{\theta} = \mu g t + V_0 + \mu g t - r\omega_0 = 2\mu g t + V_0 - r\omega_0$

$V_g(t_1) = 0 \Leftrightarrow \boxed{t_1 = \frac{r\omega_0 - V_0}{2\mu g}}$

d) Après t_1 : $V_g = 0$ → $\dot{x} = -r\dot{\theta} \Rightarrow \ddot{x} = -r\ddot{\theta} \Rightarrow \frac{T}{m} = -\frac{T}{m}$

soit $T = 0$ → $\ddot{x} = 0$ → $\dot{x} = cte = \dot{x}(t_1) = \mu g t_1 + V_0$

$\ddot{\theta} = 0$ → $\dot{\theta} = cte = \dot{\theta}(t_1) = \frac{\mu g t_1}{r} - \omega_0$

→ $\dot{x}(t) = \frac{r\omega_0 + V_0}{2} = cte$ et $\dot{\theta}(t) = cte = -\frac{r\omega_0 + V_0}{2r}$ → mvt uniforme

Partie III :

a) $W_{0\to t_1} = \int \vec{T} \cdot \vec{v_g}$ avec $v_g = \dot{x} + r\dot{\theta}$ et $\frac{dv_g}{dt} = \ddot{x} + r\ddot{\theta} = \frac{T}{m} + \frac{T}{m} = \frac{2T}{m}$

$$= \int \frac{m}{2}\frac{dv_g}{dt} \cdot \vec{v_g} = m\left[\frac{v_g^2}{2}\right]_0^{t_1} = \frac{m}{2}\left[v_g^2(t_1) - v_g^2(0)\right]$$

$$W_{0\to t_1} = -\frac{m}{4}(v_0 - r\omega_0)^2$$

b) $\Delta \mathcal{E}_k = \mathcal{E}_k(t_1) - \mathcal{E}_k(0)$

avec $\mathcal{E}_k(t_1) = \mathcal{E}_k^*(t_1) + \frac{1}{2} m v_{G/R}^2(t_1)$

$$= \frac{1}{2}(mr^2)\dot{\theta}^2(t_1) + \frac{1}{2} m \dot{x}^2(t_1)$$

$$= \frac{1}{2} m \left[r^2\left(\frac{r\omega_0 + v_0}{2r}\right)^2 + \left(\frac{r\omega_0 + v_0}{2}\right)^2 \right]$$

$$= \frac{1}{4} m (r\omega_0 + v_0)^2$$

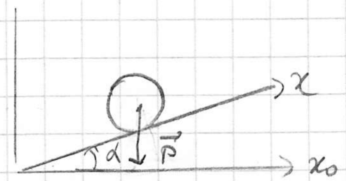
$$\mathcal{E}_k(0) = \frac{1}{2} m r^2 \omega_0^2 + \frac{1}{2} m v_0^2 = \frac{1}{2} m (r^2\omega_0^2 + v_0^2)$$

Soit $\boxed{\Delta \mathcal{E}_k = -\frac{m}{4}(r\omega_0 - v_0)^2 = W_{0\to t_1}}$

Partie IV :

a) $\vec{v_g} = \vec{V}(I_1 \in C/R) - \vec{V}(I_1 \in \text{Tapis}/R)$

$$= (\dot{x} + r\dot{\theta})\,\vec{e_x} - V_1\,\vec{e_x}$$



b) $\begin{cases} m\ddot{x} = T - mg\sin\alpha \\ m\ddot{y} = N - mg\cos\alpha = 0 \end{cases}$

$$\left.\frac{d\vec{L_G}}{dt}\right|_R = \vec{\mathcal{M}}_G(\vec{P} + \vec{T} + \vec{N}) = \vec{\mathcal{M}}_G(\vec{T} + \vec{N}) = \vec{GI} \wedge \vec{T} = rT\vec{e_z}$$

$$\to mr^2\ddot{\theta} = rT$$

c) $\frac{dv_g}{dt} = \ddot{x} + r\ddot{\theta} = \frac{T}{m} - g\sin\alpha + \frac{T}{m} = \frac{2T}{m} - g\sin\alpha$

avec $v_g \neq 0 \to \|\vec{T}\| = \mu\|\vec{N}\|$ et $v_g < 0 \to T > 0 \to T = \mu mg\cos\alpha$

Soit $\frac{dv_g}{dt} = 2g\cos\alpha - g\sin\alpha$